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IMPROVED ANALYSIS OF COULOMB-NUCLEAR INTERFERENCE EXPERIMENT FOR PIONS ON ^{16}O*

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ABSTRACT

We have reanalyzed the small angle elastic scattering data of Mutchler et al. for ^{16}O and estimate that the real part of the forward amplitude vanishes at $T_\pi = 178 \pm 4$ MeV, a result substantially higher in energy than previously reported. Our analysis is simpler than the standard one; we determine the Bethe phase and the real and imaginary amplitudes directly from the data. Our extracted Bethe phase is consistent with the theory of West and Yennie. Results for ^{12}C are also reported.

I. INTRODUCTION

In the Coulomb-nuclear interference experiment the angular distribution $d\sigma/d\Omega$ for elastically scattered pions is measured near the forward direction, where the Coulomb and strong amplitudes are expected to be of comparable size. The object of these experiments is to measure the forward scattering amplitude. In this talk we propose a new procedure for data analysis which is simpler than the conventional one.[1-3] We examine in some detail the application of the new scheme to the ^{16}O data of Mutchler et al.,[3] who provide good quality angular distributions over the angular range $5^o \leq \theta \leq 11^o$ for π^+ and π^- incident at three energies near the (3-3) resonance.

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Our method is based on two propositions. The first is that the data should be analyzed to extract the forward amplitude $f_N(0)$, where $f_N(\theta)$ is defined in terms of the complete elastic amplitude $F(\theta)$ and a Coulomb amplitude $f_C(\theta)$ by

$$f_N(\theta) = F(\theta) - f_C(\theta) \quad . \tag{1}$$

This aspect of the analysis scheme is similar to that proposed in Ref. 4 for analysis of total cross section data and should therefore be reliable even for heavy nuclei and for low-energy incident pions, where the influence of the Coulomb interaction is very great. All three of the amplitudes in Eq. (1) depend on the charge of the pion, and we will often indicate by a superscript + the amplitude for π^+ and by - that for π^-. The analysis for $f_N^{\pm}$ is examined in detail in Sec. II.

Our second point is that knowledge of $f_N^+(0)$ and $f_N^-(0)$ determines directly the purely strong amplitude $f_S(0)$. Numerous theoretical investigations have been carried out to illuminate the connection between $f_N^{\pm}$ and f_S, beginning with Bethe[5] for proton scattering from nuclei and extending to the present time. For N = Z nuclei, all such theories are consistent with the following characterization: the dominant influence of Coulomb interaction on the nuclear amplitude is by the relation

$$f_N(0) = e^{i\phi} f_S(0) \quad , \tag{2}$$

where ϕ is a complex number satisfying

$$\phi \equiv \phi^{+} = -\phi^{-} \quad , \tag{3}$$

and, of course, where isotopic spin invariance implies

$f_S(\theta)$ is independent of charge. (4)

In Sec. III we shall show how to extract $f_S(0)$ and ϕ from the data using only the general properties in Eqs. (2-4). In Refs. 1-3 the details of the theories were used as an integral part of the analysis.

It is not our purpose to justify the details of the theories which have been used to analyze the data, but rather to judge the quality of these theories relative to the experimental results. It is therefore useful to obtain expressions for ϕ in the theories. For the analysis of Refs. 1 and 2, the theory of pion-nucleon scattering developed by West and Yennie[6] was used for ϕ. In this theory the expression for ϕ is given as an integral over f_S and a Coulomb amplitude. West and Yennie show how the electromagnetic form factors of the pion and target are to be taken into account. The result of doing the integral is

$$\phi = \phi_B \equiv \gamma\left[C + \log \frac{2}{3} k^2 \left(r_S{}^2 + r_C(N)^2 + r_C(\pi)^2\right)\right] + 2\sigma_0 \quad , \tag{5}$$

where γ is the usual Coulomb parameter,[7] σ_0 the $\ell = 0$ point Coulomb phase,[†] k is the incident pion momentum, C is Euler's constant ($C \approx 0.5772$), r_S is an appropriate strong interaction radius for the nucleus, $r_C(N)$ is the nuclear charge radius, and $r_C(\pi)$ is the pion charge radius. West and Yennie point out that there are (small) model-dependent corrections to this formula. In Ref. 1 the full expression (5) was used in the analysis, but in Refs. 2 and 3 the electromagnetic radii were omitted in this expression. In this theory ϕ is often called the "Bethe phase." In the analysis of Ref. 3 a further modification was made to Eq. (2) according to the theory of Fäldt and Pilkuhn.[8] They write

$$f_N(k,0) = \frac{e^{i\phi_B}}{1 + \delta} f_S\,[k/(1 + \delta),0] \quad , \tag{6}$$

where

$$\delta = \gamma/kr_S \quad . \tag{7}$$

According to Fäldt and Pilkuhn,[8] when $\delta \neq 0$ the deviation of the pion classical trajectory from a straight line due to the Coulomb interaction has been taken into account.

[†]We add the term $2\sigma_0$ to the result of West and Yennie so that Eq. (5) corresponds to our choice of phase in the Coulomb amplitude. See Eq. (10).

To lowest order in γ[†] we have

$$\frac{1}{1+\delta} f_S\,[k/(1+\delta),0] = \frac{f_S(k,0)}{1+\delta}\left[1 - \frac{\delta k}{f_S(k,0)}\ \frac{d}{dk} f_S\,(k,0)\right]$$

$$= f_S\,(k,0)\,\exp\left[-\frac{\delta k}{f_S(k,0)}\frac{d}{dk} f_S\,(k,0) - \delta\right] , \qquad (8)$$

so as long as γ is small, Eq. (6) is consistent with Eq. (2) provided we identify

$$\phi = \phi_B + i\,\delta + i\,\frac{\delta k}{f_S(0)}\frac{d}{dk} f_S(0) \quad . \qquad (9)$$

† In this work we shall always retain terms correctly to order γ. For the energy range of interest in this work, $|\gamma| \approx 0.06 - 0.07$ for ^{16}O.

II. ANALYSIS FOR $f_N^{\pm}$

In our analysis we have taken $f_C(\theta)$ to be the point Coulomb amplitude[7]

$$f_C(\theta) = \frac{-\gamma}{2k \sin^2 \theta/2} \exp\left[-i\gamma \log \sin^2 \theta/2 + 2i\sigma_0\right] . \tag{10}$$

Because f_C is a point Coulomb amplitude, $f_N(\theta)$ contains some of the short-ranged Coulomb interaction which is conventionally[†] included in $f_C(\theta)$. To remove this from $f_N(\theta)$, it is only necessary to calculate the amplitude $\delta f_C(\theta)$,

$$\delta f_C(\theta) \equiv f_{C,\text{extended}} - f_{C,\text{point}} \quad , \tag{11}$$

and to subtract it from $f_N(\theta)$

$$f_N'(\theta) \equiv f_N(\theta) - \delta f_C(\theta) \quad . \tag{12}$$

This subtraction may be done conveniently after the proposed analysis. The quantity $f_N'(\theta)$ is the nuclear amplitude measured relative to the Coulomb amplitude for an extended charge distribution.

[†]Equation (5) is the correct expression for ϕ provided that f_N is measured relative to the Coulomb amplitude for an extended charge distribution.

The differential scattering cross section is given by

$$\frac{d\sigma}{d\Omega} \equiv |F(\theta)|^2$$
$$= |f_C(\theta)|^2 + |f_N(\theta)|^2 + 2\,\mathrm{Re}\, f_C^*(\theta)\, f_N(\theta) \quad . \tag{13}$$

Because we are interested in small angles only, we may take

$$f_N(\theta) \cong A + B \sin^2 \theta/2 \quad , \tag{14}$$

where A and B are complex constants to be determined from a fit of Eq. (13) to the data. Depending on the quality of the data, the nucleus, energy, etc., fewer or more parameters may be appropriate. In any case, the optimum number of parameters should be selected entirely on the basis of a statistical analysis of the data. For the data of Mutchler et al.[3] we have found that $f_N(\theta)$ may be taken to be constant.† In this case we retain only the A-term in Eq. (14) and find

$$\sigma(\theta) - |f_C(\theta)|^2 = |A|^2 - \frac{\gamma}{k \sin^2 \theta/2} (\mathrm{Re}\, A \cos W - \mathrm{Im}\, A \sin W) \quad , \tag{15}$$

†This is justified by the results of the standard error analysis. However, if we assume $f_N(\theta) = f_N(0) \exp[-\, r_S^2|t|/6]$ as in Ref. 3, then $f_N(\theta)$ changes by roughly 10% over the angular range $5^\circ \leq \theta \leq 11^\circ$, which is comparable to the experimental errors in $\frac{d\sigma}{d\Omega} - |f_C(\theta)|^2$.

where

$$W = \gamma \log \sin^2 \theta/2 - 2\sigma_0 \quad . \tag{16}$$

Re A and Im A were varied to obtain a minimum χ^2 fit to the data. The results we find are given in Table I. Note that the χ^2 per degree of freedom are quite reasonable and average to 0.95, which is a statistically significant improvement over the analysis of the same data in Ref. 3, Table III. It is also significant that errors on our extracted amplitudes are appreciably less than the errors on similar quantities found in Ref. 3. Since $f_N^{\pm}$ was not the object of analysis in Ref. 3, we cannot easily compare results at this stage of the analysis. The results in Table I are shown graphically in Figs. 1 and 2. Figure 1 shows Re $f_N^{\pm}(0)$ and the best-fit straight lines to the data. Figure 2 shows Im $f_N^{+}(0)$ and Im $f_N^{-}(0)$, which are the same to within statistics. The curve in Fig. 2 is an eyeball fit to the data.

III. ANALYSIS FOR f_S and ϕ_B

In order to extract the conventional Bethe phase we must make a correction for the charge form factor. In the Born approximation we easily find an expression for $\delta f_C(0)$, defined in Eq. (11)

$$\delta f_C(0) = \frac{\gamma k r_C^2}{3} \ , \tag{17}$$

where r_C^2 is the sum of the squares of the root-mean-square charge radii for the pion and nucleus,

$$r_C^2 = r_C(N)^2 + r_C(\pi)^2 \ . \tag{18}$$

If we go beyond the Born approximation, then there are small corrections to Eq. (17) (of order γ^2). The correction $\delta f_C(\theta)$ should not contain the singular phase exp $i[\gamma \log \sin^2 \theta/2]$ found in Eq. (3) of Ref. 3.† We thus have from Eq. (12)

$$f_N'(0) = f_N(0) - \frac{\gamma k r_C^2}{3} \tag{19a}$$

$$\approx f_N(0) \exp\left[-\frac{\gamma k}{3} \frac{r_C^2}{f_S(0)}\right] , \tag{19b}$$

where we have consistently retained terms only through order γ.

†This is one of two errors made in Ref. 3 in the Coulomb amplitude [see their Eq. (13)]. The other is in making the Fäldt and Pilkuhn correction in the Coulomb term. The long-range (small t) behavior of the Coulomb amplitude is known <u>exactly</u> from gauge invariance and must go like $|f_C| \sim 2k\eta/t$, <u>not</u> $2k\eta(1 + \delta)/t$.

We now have from Eqs. (2 and 19b)

$$f_N(0) = f_S(0) \exp i \left[\phi - i \frac{\gamma k}{3} \frac{r_C^2}{f_S(0)} \right] . \tag{20}$$

It follows from Eq. (20) and Eqs. (3 and 4) that

$$f_S(0) = \sqrt{f_N^+(0)\ f_N^-(0)} \tag{21a}$$

$$\phi = -i \log f_N^+(0)/f_S(0) + i \frac{\gamma k}{3} \frac{r_C^2}{f_S(0)} . \tag{21b}$$

Table II shows the extracted values of $f_S(0)$, obtained from Eq. (21a), $X \equiv -i \log f_N^{(+)}/f_S(0)$, and the values of ϕ calculated from Eq. (21b) with $r_C(N) = 2.71$ fm and $r_C(\pi) = 0.8$ fm. Figures 1 and 2 show Re $f_S(0)$ and Im $f_S(0)$ (curves S) which are calculated from fits to the curves for Re $f_N^{\pm}$ and Im $f_N^{\pm}$. We estimate that the resonance, defined as the point where Re $f_S(0)$ crosses zero, lies at T_π = 178 ± 4 MeV.

We show in Table III the values of ϕ_B calculated from the expressions in Eqs. (5 and 9). The values predicted by the West and Yennie theory are in very good agreement with experiment. Note that the addition of the term $2\sigma_o$ in Eq. (5) reduces the size of the Bethe phase substantially. The percentage disagreement between theory and experiment is somewhat smaller when the original definition of ϕ_B is used. Table III also shows the prediction of the theory of Fäldt and

Pilkuhn. We have used the West and Yennie result for ϕ_B in Eq. (9).[†] We have estimated (by numerical differentiation of the fits to the data) the correction term of Eq. (9),

$$i\,\delta + i\,\frac{\delta k}{f_S(0)}\,\frac{d}{dk}\,f_S(0) \quad , \tag{22}$$

and find that the term $i\delta$ tends to cancel against the imaginary piece of the second term in Eq. (22). The real part of Eq. (22) is positive and roughly equal to δ in magnitude. It is therefore about a 10% correction to Eq. (5). The correction of Fäldt and Pilkuhn improves somewhat the theoretical prediction at the higher energy but slightly worsens the result at the lower energies.

[†] We are told by G. West that this may involve some double counting.

IV. ANALYSIS OF ^{12}C DATA

We have also analyzed the ^{12}C data of Binon[1] and Mutchler et al.[2] We analyzed data on the angular range $5^o \leq \theta \leq 11^o$ only, in order to justify taking the nuclear amplitude to be constant; there are some inconsistencies in our results which may be due in part to this truncation of the data. The χ^2 are generally poorer than for ^{16}O (average $\chi^2/N = 1.62$). Because the data points were taken over widely spaced energy intervals, we cannot determine the position of the resonance with as much confidence. We find

$$E_R = 170 \pm 20 \text{ MeV} \quad . \tag{23}$$

For the purpose of determining the resonance position, it is useful to make measurements spaced closer in energy to the expected zero of Re f_S so that straight line fits to $f_N^{\pm}(0)$ may be expected to be good approximations.

V. DISCUSSION

In the analysis of Ref. 3 the data was analyzed in several different ways. In each scheme the amplitude $f_S(t)$ was taken to have an exponential dependence on t with a rate of fall-off related to the charge radius of the nucleus. The integral formulation of the Bethe phase was used. The total cross section was measured in a separate experiment and used to fix Im $f_S(0)$ via the optical theorem.

In the first scheme the data for π^+ and π^- were separately analyzed. The analysis incorporated Eq. (2), with the expression for ϕ_B taken from the paper by West and Yennie,[6] Eq. (5.1). The adjustable parameter for each set of data was Re $f_S(0)$, yielding a value for Re f_S^+ and Re f_S^-. The result of this analysis was that $f_S^+ \neq f_S^-$, a contradiction. The conclusion of the analysis was that there were experimental inadequacies in the data, and so a second analysis scheme was undertaken in which the <u>difference</u> $\sigma^+(\theta) - \sigma^-(\theta)$ was analyzed. It was the analysis in terms of the difference which shifted the resonance to 162 ± 3 MeV. Although the theory is in qualitative agreement with the data, it is not in quantitative agreement with the charge independence of f_S. We note here that the charge dependence of f_S in the analysis of Ref. 3 can be explained as due to an incorrect choice for φ_B: we have already remarked that the expression used by Mutchler et al.[3] did not take account of the electromagnetic form factors according to Eq. (30) of the paper by West and Yennie. We believe that this incorrect value of ϕ_B accounts in part for the disagreement between our results for the position of the resonance.

In our analysis we have turned the problem around. We analyze both π^+ and π^- data together and thereby are able to extract ϕ_B and $f_S(0)$ without having to assume any form for $f_S(t)$. We were surprised to see the accuracy to which these experiments determine the total cross section. The extracted value of $f_S(0)$ is not subject to imperfections of any model of the strong interaction; the extracted value of ϕ_B may be used to test the theory of the Bethe phase. As far as we know the limit of validity of the Bethe phase has not been set experimentally; we feel that one should be especially cautious of the theory for high-Z targets and low-energy projectiles. We hope that the methods we have developed will stimulate more theoretical work on the relation between the strong and Coulomb amplitudes.

We thank several members of the Rice group for useful discussions and for pointing out a numerical error in a preliminary calculation. We also thank the Rice group for providing their data for $^{12}C\ \pi^+$ scattering.

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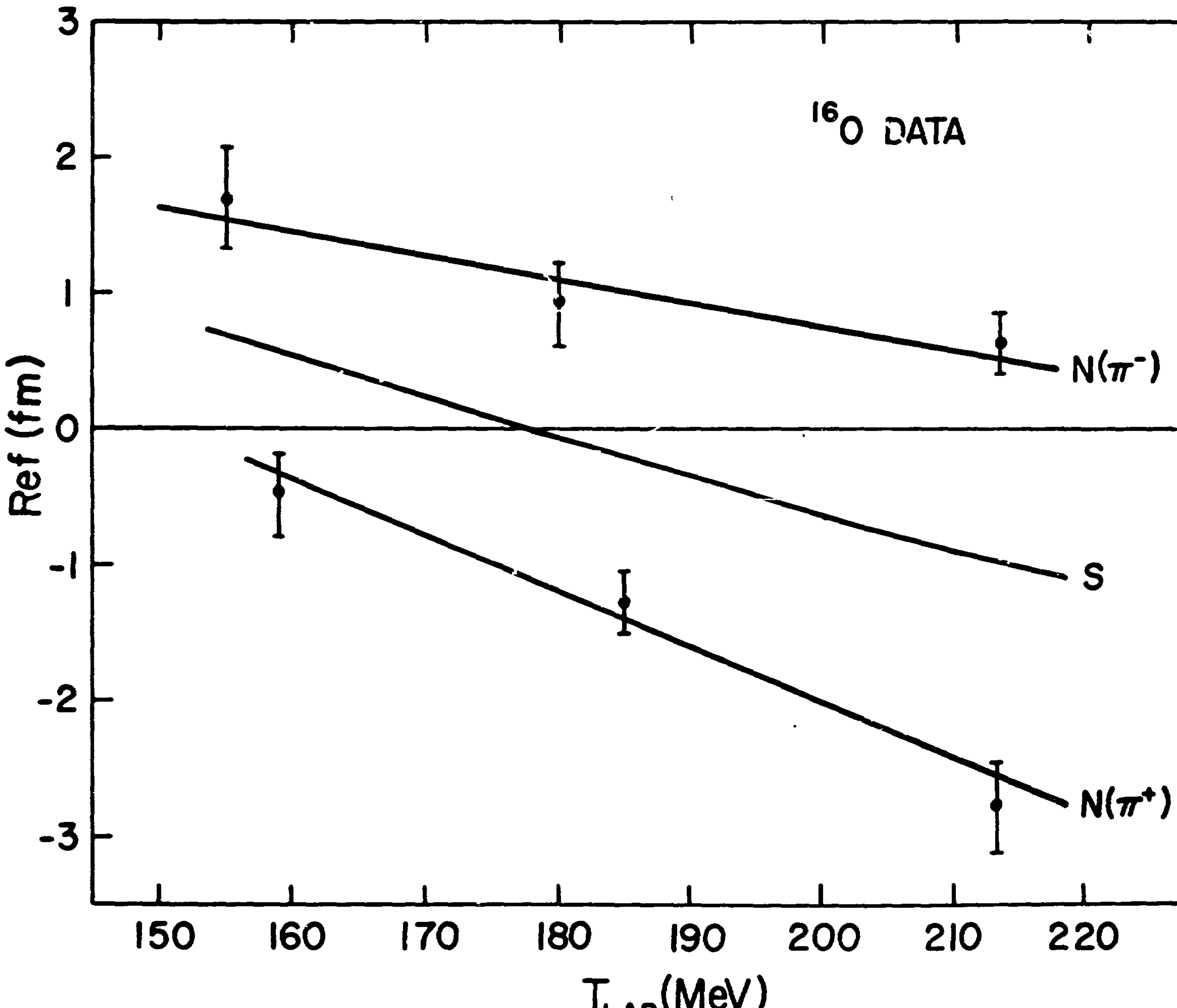
FIGURE CAPTIONS

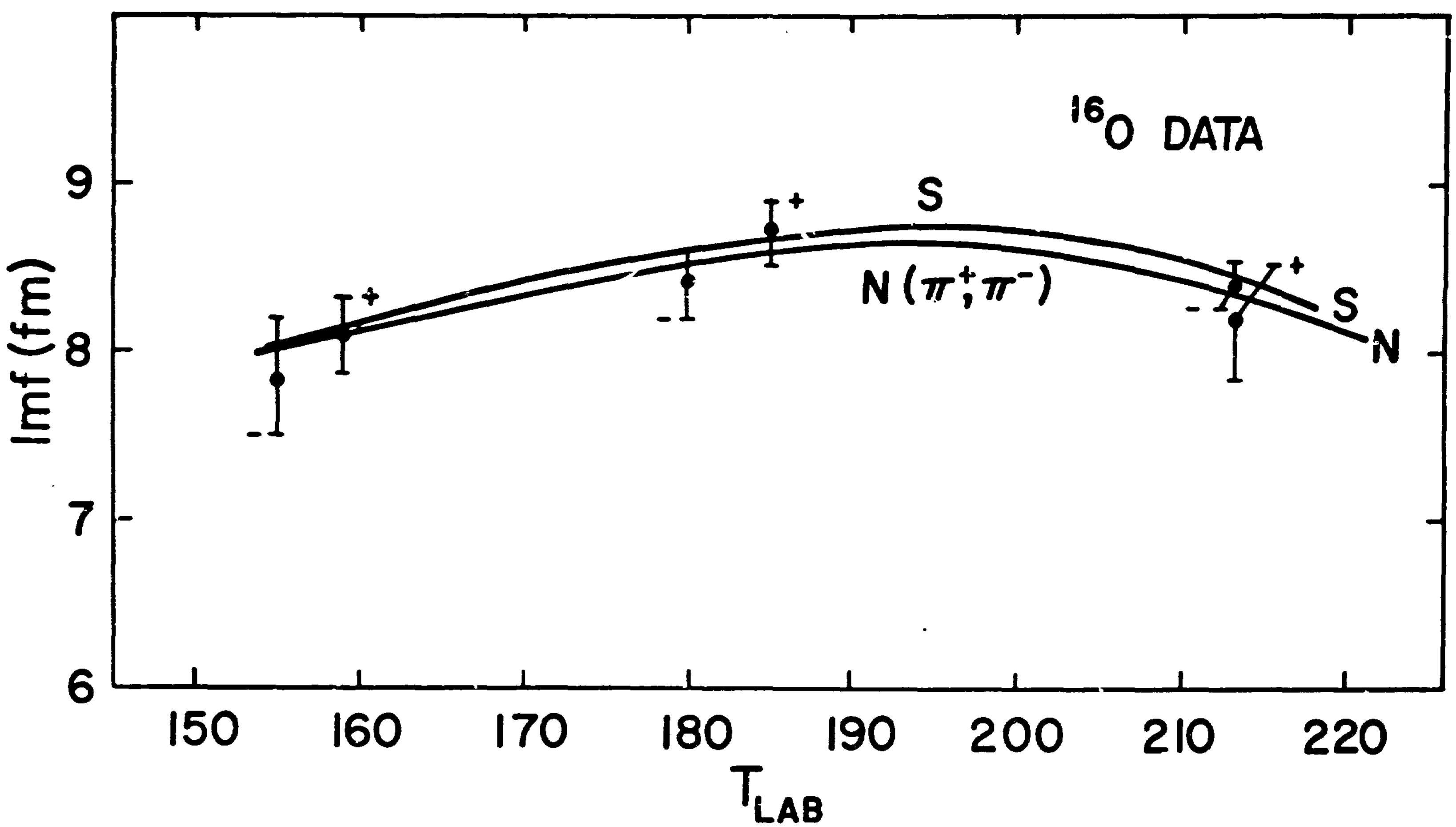
Fig. 1. Values for Re $f_N^{\pm}(0)$ (curve N) and Re $f_S(0)$ (curve S).

Fig. 2. Values for Im $f_N^{\pm}(0)$ (curve N) and Im $f_S(0)$ (curve S).



3
2
1
0
-1
-2
-3
Ref (fm)
^{16}O DATA
$N(\pi^-)$
S
$N(\pi^+)$
150
160
170
180
190
200
210
220
T_{LAB}(MeV)

Figure 1



^{16}O DATA
S
N ($\pi^+;\pi^-$)
S
N
+
+
+
–
–
–
9
8
7
6
Imf (fm)
150
160
170
180
190
200
210
220
T_{LAB}

Figure 2

TABLE I

ANALYSIS OF ^{16}O DATA[a]

	π^+			π^-		
T_π (MeV)	159	185	213	155	180	213
χ^2/N	1.19	0.75	1.53	1.04	0.53	0.68
Re A (fm)	-0.49	-1.28	-2.80	1.69	0.91	0.62
δ Re A (fm)	±0.31	±0.24	±0.35	±0.37	±0.31	±0.23
Im A (fm)	8.09	8.71	8.18	7.83	8.40	8.40
δ Im A (fm)	±0.22	±0.18	±0.33	±0.34	±0.21	±0.14

[a] T_π is the laboratory kinetic energy of the pion; A is $f_N(0)$ in the laboratory frame.

TABLE II

EXTRACTED VALUES OF f_S AND BETHE PHASE

T_π	Re $f_S(0)$	Im $f_S(0)$	Re χ^a	Im χ^a	Re ϕ_B	Im ϕ_B
160	0.54	8.16	0.12	0.0074	0.14	0.0087
170	0.22	8.41	0.13	0.0036	0.15	0.0045
180	-0.04	8.57	0.14	-0.001	0.16	-0.0011
190	-0.34	8.72	0.15	-0.006	0.18	-0.0075
200	-0.64	8.71	0.16	-0.011	0.19	-0.0145
210	-0.90	8.54	0.17	-0.018	0.20	-0.0220

[a] $\chi = -i \log f_N^+(0)/f_S(0)$

TABLE III

COMPARISON OF EXPERIMENTAL AND THEORETICAL VALUES OF BETHE PHASE

T_π	ϕ_B (W-Y)	ϕ_B (F-P)	ϕ_B (EXPT)
160	0.15	0.17	0.14
170	0.16	0.18	0.15
180	0.16	0.18	0.16
190	0.16	0.18	0.18
200	0.17	0.19	0.19
210	0.17	0.18	0.20